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## Liquid Crystals

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## The torque exerted on a surface-stabilised ferroelectric liquid crystal cell in an electric field

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The hydrodynamic equations for surface-stabilised ferroelectric liquid crystal cells in the presence of an electric field are calculated. The components of velocity are obtained. Then the torque exerted on the torsional oscillator in which the cell is contained is obtained. We also determine the changes in the resonant frequency as a function of the electric field.

**Keywords:** ferroelectric liquid crystal; Fredericks transition; Torsional oscillator; stress tensor

### 1. Introduction

Smectic liquid crystals are systems which are positionally disordered but reveal a long-range orientational order. This property is described on a mesoscopic level by a unit vector field  $\hat{\mathbf{n}}(\mathbf{r})$ , which is called the director. The experimentally observed rotation of smectic liquid crystal layers, under the action of external electric fields, has attracted a significant amount of interest. This unusual effect is observed only in chiral smectic phases and is believed to be rotated by the electroclinic effect (1, 2).

To the best of the authors' knowledge, there have been only a few papers published on the dynamics of Sm- $C^*$  liquid crystals. Particle dispersion in liquid crystals has been studied. The interactions between liquid crystals and particle dispersions were investigated in (3). Recently, the manipulation of colloidal particles was studied in (4). The backflows in the Goldstone mode dielectric response of chiral Sm- $C^*$  has been studied by Carlsson (5). A Fredericks transition due to the presence of the spontaneous polarisation in Sm- $C^*$  has been predicted by Zimmermann *et al.* (6). The possibility of inducing a Fredericks transition in a wedge stability has been discussed by Carlsson *et al.* (7). Zou *et al.* have derived the flow-coupled switching equations for surface-stabilised ferroelectric liquid crystal cell (SSFLC) devices (8). Here, the dynamic equations of SSFLCs are calculated and then the components of velocity are derived. Finally, the torque exerted on the cell in the presence of an electric field is computed.

The aim of our paper is to present the calculation of the torque exerted on a SSFLC in an electric field. The outline of the paper is as follows. In section 2, we

obtain the total free energy and then discuss the threshold field in which the Fredericks transition will occur. In section 3, we calculate the dynamic equations and obtain the components of velocity. Finally, in section 4, we compute the torque exerted on the cell.

### 2. Euler equations and the Fredericks transition

Consider a slab of ferroelectric liquid crystal (Sm- $C^*$ ) inside a torsional oscillator, of width  $d$ , placed in an electric field  $\mathbf{E}$ . We assume that the orbital texture is not affected by the flow associated with the motion of the torsional oscillator. In this work, we assume that the smectic layers consist of uniform planes with a fixed orientation parallel to the  $x$ - $y$  plane (see Figure 1). We call a unit vector  $\mathbf{a}$  normal to the smectic layer. Each layer is normal to the slab surfaces, thus  $\mathbf{a}$  is parallel to the  $z$ -axis, (see Figure 1). We assume that the system studied is free from dislocation of constant layer thickness; hence, the layer normal  $\mathbf{a}$  must fulfill the constraint (9, 10):

$$\nabla \times \mathbf{a} = 0. \quad (1)$$

The director field,  $\mathbf{n}$ , makes an angle  $\theta$  with the  $z$ -axis. The projection of the director field into the smectic planes ( $x$ - $y$  plane) is called a unit vector  $\mathbf{c}$  ( $c$ -director). In order to describe the orientation of the  $c$ -director, we introduce azimuthal angle,  $\phi$ , which is the angle between the  $c$ -director and the  $x$ -axis:

$$\mathbf{c} = (\cos \phi, \sin \phi, 0). \quad (2)$$

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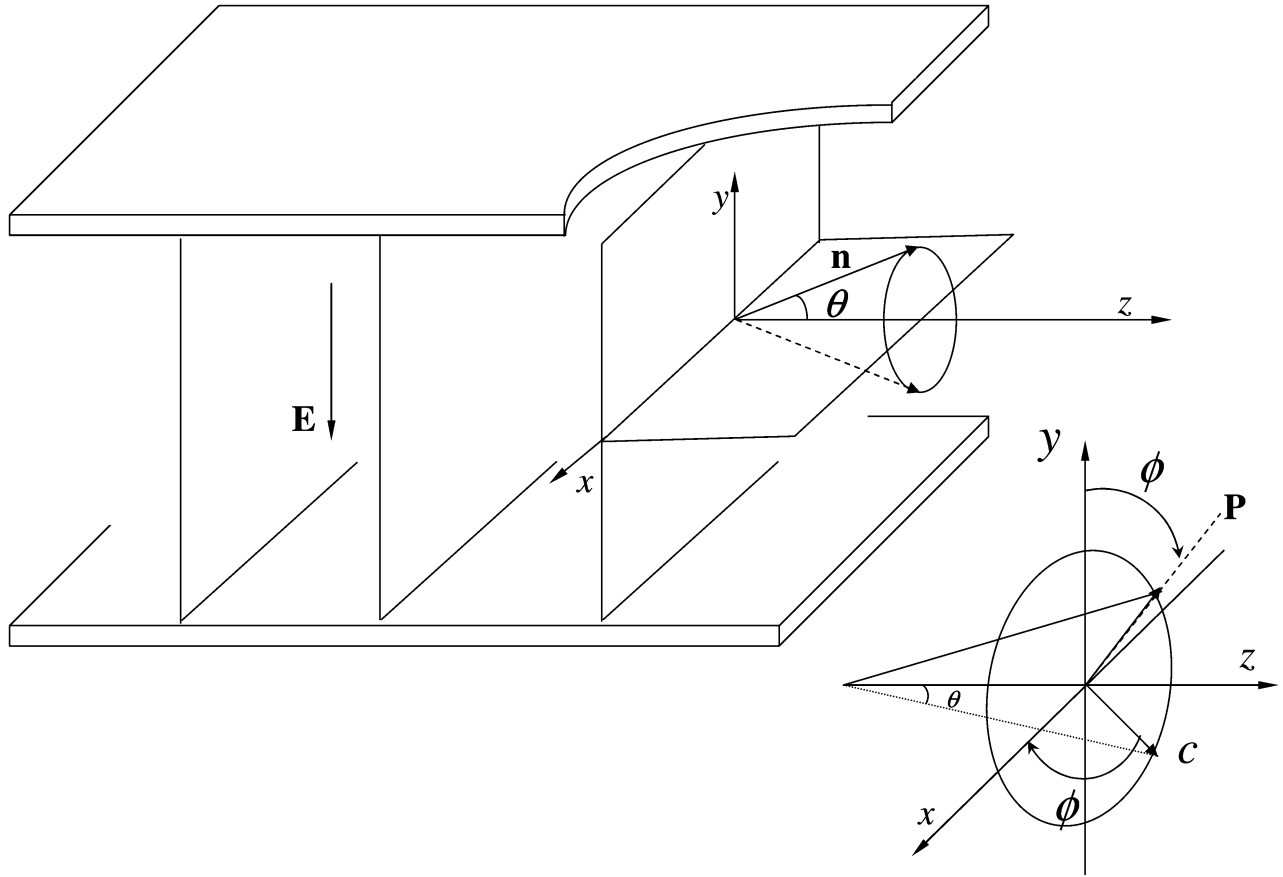


Figure 1. The bookshelf geometry of a SSFLC cell.

Introducing a unit vector  $\mathbf{b}$  according to

$$\mathbf{b} = \mathbf{a} \times \mathbf{c}, \quad (3)$$

this unit vector will coincide with the polarisation vector ( $\mathbf{P} = P\mathbf{b}$ ). Without applying an external field, the ground state has a homogeneous structure. At any given field,  $\mathbf{E} = -E\mathbf{j}$ , the stable equilibrium state can be found by minimising the total free energy with respect to variations in the  $c$ -director. In the one constant approximation, and neglecting the dielectric coupling, the total density of free energy is written as (5, 12)

$$f = \frac{B}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + PE \cos \phi. \quad (4)$$

The first term comes from the Frank free energy, whereas the last term comes from the electric field;  $B$  is the elastic constant and  $P$  is spontaneous electric polarisation, which lies in the smectic planes perpendicular to  $\mathbf{a}$  and  $\mathbf{c}$ .

At  $E = E_c$  the Fredericks transition will be induced by forcing the polarisation to point downwards

everywhere except at the boundaries of the cell (because of strong anchoring at the bounding plates). The value of  $\phi(y)$  at the critical field can be found by minimising the total free energy. By using the Euler–Lagrange equation and small values of  $\phi(y)$ , we have

$$\left( \frac{dy}{d\phi} \right)^2 = \left( \frac{B}{PE} \right) \frac{1}{\phi_m^2 - \phi^2}. \quad (5)$$

The solution to (5) is

$$\phi(y) = \phi_m \cos \left( \frac{\pi}{d} y \right), \quad (6)$$

where  $\phi_m$  is determined variationally.

Now, we calculate the threshold field from (5). By performing the integrals of (5), from  $\phi = \phi_m$  to  $\phi = 0$  and  $y = 0$  to  $y = d/2$ , the threshold field is given as

$$E_c = \frac{\pi}{d} \sqrt{\frac{B}{P}}. \quad (7)$$

To calculate  $\phi_m$ , we first calculate the total free energy of the cell

$$F = \frac{B\pi^2 E}{d E_c} \left\{ \frac{\phi_m^4}{64} + \frac{\phi_m^2}{4} \left( \frac{E_c}{E} - 1 \right) + 1 \right\}. \quad (8)$$

The value of  $\phi_m$  can be found by minimising the total free energy with respect to the amplitude  $\phi_m$  for a fixed value of applied electric field  $\mathbf{E}$ . This gives

$$\phi_m = 0, \quad (9)$$

or

$$\phi_m = \sqrt{8 \left( 1 - \frac{E_c}{E} \right)}, \quad (10)$$

where the solution  $\phi_m=0$  is for  $E < E_c$ . It is obvious that in this case, we do not have any texture in the system.

### 3. The hydrodynamics equations of a narrow slab of Sm-C\* liquid crystal in a torsional oscillator

We now proceed by writing down the equations governing the hydrodynamic behavior of the Sm-C\* phase. The basic mathematical formulation of these equations has been derived by Leslie, Stewart and Nakagawa (LSN) (9), and has been interpreted further and reformulated by Zou *et al.* (8). The LSN theory has 13 unknowns and, with 13 coupled partial differential equations, is a complex system of equations to solve without simplification. We use some approximations which are explained in the following.

We assume that the total density of fluid remains constant during the oscillator motion. Then conservation of linear momentum requires that (12)

$$\rho \dot{v}_i = F_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tilde{t}_{ij}}{\partial x_j}, \quad (11)$$

where  $\rho$  is the density of liquid crystal,  $F_i$  the sum of external forces,  $p$  is the usual pressure and  $\tilde{t}_{ij}$  is the viscous part of stress tensor. We restrict our attention to small tilt angle  $\theta$ . Thus, the dynamics equation that arise from the angular momentum balance are self-satisfied. The viscous part of the stress tensor,  $\tilde{t}_{ij}$ , is most conveniently expressed as the sum of its symmetrical,  $\tilde{t}_{ij}^s$ , and antisymmetrical,  $\tilde{t}_{ij}^a$ , parts

$$\tilde{t}_{ij} = \tilde{t}_{ij}^s + \tilde{t}_{ij}^a. \quad (12)$$

By introducing the following quantities

$$D_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad W_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} \right), \quad (13)$$

$$D_i^a = D_{ij} a_j, \quad D_i^c = D_{ij} c_j,$$

$$A_i = \dot{a}_i - W_{ik} a_k, \quad C_i = \dot{c}_i - W_{ik} c_k,$$

we can write the viscous stress tensors as (9),

$$\begin{aligned} \tilde{t}_{ij}^s = & \mu_0 D_{ij} + \mu_1 a_p D_p^a a_i a_j + \mu_2 (D_i^a a_j + D_j^a a_i) \\ & + \mu_3 c_p D_p^c c_i c_j + \mu_4 (D_i^c c_j + D_j^c c_i) \\ & + \mu_5 c_p D_p^a (a_i c_j + a_j c_i) + \lambda_1 (A_i a_j + A_j a_i) \\ & + \lambda_2 (C_i c_j + C_j c_i) + \lambda_3 c_p A_p (a_i c_j + a_j c_i) \\ & + \kappa_1 (D_i^a c_j + D_j^a c_i + D_i^c a_j + D_j^c a_i) \\ & + \kappa_2 [a_p D_p^a (a_i c_j + a_j c_i) + 2a_p D_p^c a_i a_j] \\ & + \kappa_3 [c_p D_p^c (a_i c_j + a_j c_i) + 2a_p D_p^c c_i c_j] \\ & + \tau_1 (C_i a_j + C_j a_i) + \tau_2 (A_i c_j + A_j c_i) \\ & + 2\tau_3 c_p A_p a_i a_j + 2\tau_4 c_p A_p c_i c_j, \end{aligned} \quad (14)$$

and

$$\begin{aligned} \tilde{t}_{ij}^a = & \lambda_1 (D_j^a a_i - D_i^a a_j) + \lambda_2 (D_j^c c_i - D_i^c c_j) \\ & + \lambda_3 c_p D_p^a (a_i c_j - a_j c_i) + \lambda_4 (A_j a_i - A_i a_j) \\ & + \lambda_5 (C_j c_i - C_i c_j) + \lambda_6 c_p A_p (a_i c_j - a_j c_i) \\ & + \tau_1 (D_j^a c_i - D_i^a c_j) + \tau_2 (D_j^c a_i - D_i^c a_j) \\ & + \tau_3 a_p D_p^a (a_i c_j - a_j c_i) + \tau_4 c_p D_p^c (a_i c_j - a_j c_i) \\ & + \tau_5 (A_j c_i - A_i c_j + C_j a_i - C_i a_j), \end{aligned} \quad (15)$$

where the 20 coefficients  $\mu_i$ ,  $\lambda_i$ ,  $\kappa_i$ ,  $\tau_i$  in the above equations are the viscosity coefficients of the liquid crystal. These coefficients can be classified as belonging to one of four groups. The  $\mu_0$  term relates to isotropic part of stress tensor. The four terms connected to the constant  $\mu_1$ ,  $\mu_2$ ,  $\lambda_1$  and  $\lambda_4$  are present in the Sm-A\* phase. The four terms involving the coefficients  $\lambda_2$ ,  $\lambda_5$ ,  $\mu_3$  and  $\mu_4$  relate to the nematic-like phase. Finally, there are 11 terms associated with the coefficients  $\lambda_3$ ,  $\lambda_6$ ,  $\mu_5$ ,  $\kappa_1$ ,  $\kappa_2$ ,  $\kappa_3$ ,  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ ,  $\tau_4$  and  $\tau_5$ . These coefficients depend on the tilt angle  $\theta$ . Keeping the layer normal  $\mathbf{a}$  unchanged and at the same time changing the tilt angle  $\theta$  to  $-\theta$  and  $\mathbf{c}$  to  $-\mathbf{c}$  provides a symmetric operation of the system. As a consequence, the stress tensor must be invariant under this operation. Thus, we should just keep  $\mu_i$  and  $\lambda_i$  coefficients to retain the above symmetry operation

in the system. It should be noted that the above symmetry conditions reduce the theory to that of materials having the symmetry of the classical smectic  $C_M$  phase which was proposed and introduced by McMillan (11).

We now proceed to solve the hydrodynamic equation of a narrow slab of  $Sm-C^*$  in a torsional oscillator with small oscillation amplitudes to obtain the space and time dependence of the velocity field. When studying the flow properties of the system, the liquid crystal is considered to be incompressible and the velocity field  $\mathbf{v}$  is subjected to the constraint (10)

$$\nabla \cdot \mathbf{v} = 0. \tag{16}$$

The pressure,  $p$ , would be dependent on all  $x, y, z$  and  $\phi$ . However, when the cell is not sealed, then the pressure will be released before it builds up. Therefore, it is a good approximation to assume that  $p$  is  $y$  dependent for an open edge cell. Thus  $p$  only shows up on the  $y$  component of (11). As we mentioned previously, the oscillation amplitudes are assumed to be small. On the other hand, we neglect the possibility of transportation of material between the smectic layers,  $\mathbf{a} \cdot \mathbf{v} = 0$ ; thus, the velocity field is parallel to the smectic layers everywhere, that is,  $v_z = 0$ , so we assume that the velocity field has the following form (14)

$$\mathbf{v} = [v_x(y, z)\mathbf{i} + v_y(x, z)\mathbf{j}]e^{i\omega_0 t}, \tag{17}$$

where  $\omega_0$  is the oscillation frequency of the cell.

By using (17) in (13), the non-zero components of the quantities are given as follows

$$\begin{aligned} D_{yx} &= D_{xy} = \frac{1}{2} \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right], \quad D_{zx} = D_{xz} = \frac{1}{2} \left[ \frac{\partial v_x}{\partial z} \right], \\ D_{yz} &= D_{zy} = \frac{1}{2} \left[ \frac{\partial v_y}{\partial z} \right], \quad W_{xy} = -W_{yx} = \frac{1}{2} \left[ \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \right], \\ W_{xz} &= -W_{zx} = \frac{1}{2} \left[ \frac{\partial v_x}{\partial z} \right], \quad W_{yz} = -W_{zy} = \frac{1}{2} \left[ \frac{\partial v_y}{\partial z} \right], \\ D_x^a &= \frac{1}{2} \left[ \frac{\partial v_x}{\partial z} \right], \quad D_y^a = \frac{1}{2} \left[ \frac{\partial v_y}{\partial z} \right], \\ D_x^c &= \frac{1}{2} \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right] \sin \phi, \quad D_y^c = \frac{1}{2} \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right] \cos \phi, \tag{18} \\ D_z^c &= \frac{1}{2} \frac{\partial v_x}{\partial z} \cos \phi + \frac{1}{2} \frac{\partial v_y}{\partial z} \sin \phi, \quad A_x = -\frac{1}{2} \frac{\partial v_x}{\partial z}, \\ A_y &= -\frac{1}{2} \frac{\partial v_y}{\partial z}, \quad C_x = -\frac{1}{2} \left[ \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \right] \sin \phi - v_y \sin \phi \frac{\partial \phi}{\partial y}, \\ C_y &= \frac{1}{2} \left[ \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \right] \cos \phi + v_y \cos \phi \frac{\partial \phi}{\partial y}, \\ C_z &= +\frac{1}{2} \frac{\partial v_x}{\partial z} \cos \phi + \frac{1}{2} \frac{\partial v_y}{\partial z} \sin \phi. \end{aligned}$$

By inserting (14), (15), (17), and (18) into (11) we obtain the following equations for the small angle  $\phi(y)$

$$\begin{aligned} \rho i \omega_0 v_x + \rho v_y \frac{\partial v_x}{\partial y} &= \alpha_1 \frac{\partial^2 v_y}{\partial x^2} \phi(y) + \alpha_2 \frac{\partial^2 v_x}{\partial y^2} \\ &+ \alpha_3 v_y \frac{d^2 \phi}{dy^2} + \alpha_4 \frac{\partial^2 v_y}{\partial z^2} \phi + \alpha_5 \frac{\partial^2 v_x}{\partial z^2}, \end{aligned} \tag{19}$$

$$\begin{aligned} \rho i \omega_0 v_y + \rho v_x \frac{\partial v_y}{\partial x} &= -\frac{dp(y)}{dy} + \alpha_6 \frac{\partial^2 v_y}{\partial x^2} + \alpha_7 \frac{\partial v_y}{\partial x} \frac{d\phi(y)}{dy} \\ &+ \alpha_8 \frac{\partial^2 v_y}{\partial z^2} + \alpha_9 \frac{\partial^2 v_x}{\partial z^2} \phi(y) \\ &+ \alpha_{10} \left( \frac{\partial^2 v_x}{\partial y^2} \phi(y) + \frac{\partial v_x}{\partial y} \frac{d\phi(y)}{dy} \right), \end{aligned} \tag{20}$$

and

$$0 = \frac{\partial^2 v_y}{\partial x \partial z} \phi(y) + \frac{\partial^2 v_x}{\partial y \partial z} \phi(y) + \frac{\partial v_x}{\partial z} \frac{d\phi(y)}{dy}, \tag{21}$$

where the constants  $\alpha_i$  are defined in Appendix A. These are the hydrodynamic motion equations of the system.

As we mentioned previously, we are considering the state of the smectic liquid crystal in a torsional oscillator which is slightly deformed. On the basis of this assumption and the symmetry of the problem, it is possible to use the following approximation for the velocity components and pressure

$$v_x(y, z) = \sum_{n=0}^{\infty} L_n(y) M_n(z) \phi_m^n, \tag{22}$$

$$v_y(x, z) = \sum_{n=0}^{\infty} F_n(x) G_n(z) \phi_m^n, \tag{23}$$

$$v_z = 0, \tag{24}$$

$$p(y) = \sum_{n=0}^{\infty} p_n(y) \phi_m^n. \tag{25}$$

By inserting above equations into (19), (20), and (21),

we have

$$\begin{aligned} & \rho i \omega_0 \sum_{n=0}^{\infty} L_n(y) M_n(z) \phi_m^n \\ & + \rho \sum_{n=0}^{\infty} F_n(x) G_n(z) \phi_m^n \sum_{k=0}^{\infty} \frac{dL_k(y)}{dy} M_k(z) \phi_m^k \\ & = \alpha_1 \phi(y) \sum_{n=0}^{\infty} \frac{d^2 F_n(x)}{dx^2} G_n(z) \phi_m^n \\ & + \alpha_2 \sum_{n=0}^{\infty} \frac{d^2 L_n(y)}{dy^2} M_n(z) \phi_m^n \\ & + \alpha_3 \sum_{n=0}^{\infty} F_n(x) G_n(z) \frac{d^2 \phi(y)}{dy^2} \phi_m^n \\ & + \alpha_4 \phi(y) \sum_{n=0}^{\infty} F_n(x) \frac{d^2 G_n(z)}{dz^2} \phi_m^n \\ & + \alpha_5 \sum_{n=0}^{\infty} L_n(y) \frac{d^2 M_n(z)}{dz^2} \phi_m^n, \end{aligned} \tag{26}$$

$$\begin{aligned} & \rho i \omega_0 \sum_{n=0}^{\infty} F_n(x) G_n(z) \phi_m^n \\ & + \rho \sum_{n=0}^{\infty} L_n(y) M_n(z) \phi_m^n \sum_{k=0}^{\infty} \frac{dF_k(x)}{dx} G_n(z) \phi_m^k \\ & = - \sum_{n=0}^{\infty} \frac{dp_n(y)}{dy} \phi_m^n + \alpha_6 \sum_{n=0}^{\infty} \frac{d^2 F_n(x)}{dx^2} G_n(z) \phi_m^n \\ & + \alpha_7 \frac{d\phi}{dy} \sum_{n=0}^{\infty} \frac{dF_n(x)}{dx} G_n(z) \phi_m^n \\ & + \alpha_8 \sum_{n=0}^{\infty} F_n(x) \frac{d^2 G_n(z)}{dz^2} \phi_m^n \\ & + \alpha_9 \phi(y) \sum_{n=0}^{\infty} L_n(y) \frac{d^2 M_n(z)}{dz^2} \phi_m^n \\ & + \alpha_{10} \phi(y) \sum_{n=0}^{\infty} \frac{d^2 L_n(y)}{dy^2} M_n(z) \phi_m^n \\ & + \alpha_{10} \frac{d\phi}{dy} \sum_{n=0}^{\infty} \frac{dL_n(y)}{dy} M_n(z) \phi_m^n, \end{aligned} \tag{27}$$

and

$$\begin{aligned} & \phi(y) \sum_{n=0}^{\infty} \frac{dF_n(x)}{dx} \frac{dG_n(z)}{dz} \phi_m^n \\ & + \phi(y) \sum_{n=0}^{\infty} \frac{dL_n(y)}{dy} \frac{dM_n(z)}{dz} \phi_m^n \\ & + \frac{d\phi(y)}{dy} \sum_{n=0}^{\infty} L_n(y) \frac{dM_n(z)}{dz} \phi_m^n = 0. \end{aligned} \tag{28}$$

The zero order terms of (26), (27), and (28) gives

$$\begin{aligned} & \rho i \omega_0 L_0(y) M_0(z) + \rho F_0(x) G_0(z) \frac{dL_0(y)}{dy} M_0(z) \\ & = \alpha_2 \frac{d^2 L_0(y)}{dy^2} M_0(z) + \alpha_5 L_0(y) \frac{d^2 M_0(z)}{dz^2}, \end{aligned} \tag{29}$$

$$\begin{aligned} & \rho i \omega_0 F_0(x) G_0(z) + \rho L_0(y) M_0(z) \frac{dF_0(x)}{dx} G_0(z) \\ & = - \frac{dp_0(y)}{dy} + \alpha_6 \frac{d^2 F_0(x)}{dx^2} G_0(z) + \alpha_8 F_0(x) \frac{d^2 G_0(z)}{dz^2}. \end{aligned} \tag{30}$$

It is clear that (28) does not have zero-order terms.

Owing to vanishing  $v_z$ , the boundary condition in torsional oscillator at  $y = \pm d/2$  is defined as

$$\mathbf{v} = \omega z \hat{\mathbf{i}}, \tag{31}$$

where  $\omega$  is an angular velocity of the liquid crystal in the surfaces of the cell.

The right-hand side of (29) is a function of variables  $y$  and  $z$ , whereas the left-hand side of it is a function of all three variables  $x$ ,  $y$ , and  $z$ . To overcome this difficulty one assumes  $F_0(x)G_0(z) = c_0 = \text{constant}$ . By applying the boundary condition (31) on  $c_0$  we find that  $c_0$  is zero. Thus, the zero-order pressure is constant and the zero-order velocity is along the  $x$ -axis:

$$F_0(x)G_0(z) = 0. \tag{32}$$

By substituting (32) into (29), we may write

$$\alpha_2 \frac{1}{L_0} \frac{d^2 L_0(y)}{dy^2} + \alpha_5 \frac{1}{M_0(z)} \frac{d^2 M_0(z)}{dz^2} = \rho i \omega_0. \tag{33}$$

By using the boundary condition on  $M_0(z)$  ( $M_0(z) = z$ ), we have

$$v_{0x}(y, z) = L_0(y)M_0(z) = \frac{z\omega}{\cosh(\gamma d/2)} \cosh(\gamma y), \tag{34}$$

where  $\gamma^2 = \rho i \omega_0 / \alpha_2$ .

The first-order terms of (26) and (27) give

$$\begin{aligned} & \rho i \omega_0 L_1(y) M_1(z) \\ & + \rho F_1(x) G_1(z) \frac{z\omega\gamma}{\cosh(\gamma d/2)} \sinh(\gamma y) \\ & = \alpha_2 \frac{d^2 L_1(y)}{dy^2} M_1(z) + \alpha_5 L_1(y) \frac{d^2 M_1(z)}{dz^2}, \end{aligned} \tag{35}$$

and

$$\begin{aligned} &\rho i \omega_0 F_1(x) G_1(z) + \rho \frac{z\omega}{\cosh(\gamma d/2)} \cosh(\gamma y) \frac{dF_1(x)}{dx} G_1(z) \\ &= -\frac{dp_1(y)}{dy} + \alpha_6 \frac{d^2 F_1(x)}{dx^2} G_1(z) \\ &\quad + \alpha_8 F_1(x) \frac{d^2 G_1(z)}{dz^2} \\ &\quad + \alpha_{10} \left[ \frac{z\omega \gamma^2}{\cosh(\gamma d/2)} \cosh(\gamma y) \cos\left(\frac{\pi}{d}y\right) \right. \\ &\quad \left. - \frac{z\omega \gamma \pi}{d \cosh(\gamma d/2)} \sinh(\gamma y) \sin\left(\frac{\pi}{d}y\right) \right], \end{aligned} \tag{36}$$

respectively, it is noted that the average of the fourth and fifth terms of the right-hand side of (36) are zero, that is,

$$\begin{aligned} &\frac{\alpha_{10} z \omega \gamma}{d \cosh(\gamma d/2)} \int_{-d/2}^{d/2} \left[ \gamma \cosh(\gamma y) \cos\left(\frac{\pi}{d}y\right) \right. \\ &\quad \left. - \frac{\pi}{d} \sinh(\gamma y) \sin\left(\frac{\pi}{d}y\right) \right] dy = 0. \end{aligned} \tag{37}$$

The left-hand side of (36) is a function of all three variables  $x$ ,  $y$ , and  $z$ , whereas the right-hand side is the function of variables  $x$  and  $z$ . To overcome this difficulty, one may assume  $F_1(x)G_1(z)=c_1=\text{constant}$ . By using the boundary condition, we find that the velocity is along the  $x$ -axis. Thus, we have

$$F_1(x)G_1(z)=0, \tag{38}$$

and the first-order pressure is constant. By substituting (38) into (35), we may write

$$\alpha_2 \frac{1}{L_1} \frac{d^2 L_1(y)}{dy^2} + \alpha_5 \frac{1}{M_1(z)} \frac{d^2 M_1(z)}{dz^2} = \rho i \omega_0. \tag{39}$$

By using the boundary condition on  $M_1(z)$ , ( $M_1(z)=z$ ), we have

$$v_{1x}(y, z) = L_1(y)M_1(z) = \frac{z\omega}{\cosh(\gamma d/2)} \cosh(\gamma y). \tag{40}$$

Finally, by continuing this procedure, the velocity field components may be written as

$$v_x(y, z) = \sum_{n=0}^{\infty} \frac{z\omega}{\cosh(\gamma d/2)} \cosh(\gamma y) \phi_m^n, \tag{41}$$

$$v_y(x, z) = 0. \tag{42}$$

#### 4. Calculation of the torque exerted on the slab and resonance frequency

The torque exerted by the fluid on the oscillator is defined by

$$\Gamma_y = \rho \frac{d}{dt} \int (z v_x - x v_z) dV. \tag{43}$$

Furthermore, we define  $\Delta\Gamma_y = \Gamma_y(\phi) - \Gamma_y(0)$ , which is the variation in the exerted torque due to the appearance of the texture. In the case of  $E < E_c$ , we do not have any texture; hence,  $\Delta\Gamma_y = 0$ .

By inserting (40) into (43) and after straightforward calculations we have

$$\Delta\Gamma_y = \frac{\rho R^4 \pi d \omega}{2\gamma} \frac{d\omega}{dt} \tanh\left(\frac{\gamma d}{2}\right) \sum_{n=0}^{\infty} \phi_m^n, \tag{44}$$

where  $R$  is the radius of the oscillating slab. We also write  $\Delta\Gamma_y$  in terms of dimensionless quantities  $\Delta f_1$  and  $\Delta f_2$  as (14)

$$\Delta\Gamma_y = \frac{\rho R^4 \pi d \omega}{2\gamma} \frac{d\omega}{dt} \tanh\left(\frac{\gamma d}{2}\right) (\Delta f_1 + i \Delta f_2), \tag{45}$$

where  $\Delta f_1$  and  $\Delta f_2$  are related to the inertial and dissipative effects, respectively. Here, we calculated  $\Delta f_1$  and  $\Delta f_2$  for the case in which  $\gamma d$  is much smaller than one. The amplitude of  $\phi_m$  is very small, hence

$$\Delta\Gamma_y \cong \frac{\rho R^4 \pi d d\omega}{4} \frac{d\omega}{dt} \phi_m. \tag{46}$$

As is obvious,  $\Delta f_2 = 0$  for this case and

$$\Delta f_1 \cong \phi_m = 2\sqrt{2} \left(1 - \frac{E_c}{E}\right)^{1/2}. \tag{47}$$

The motion equation of the oscillator is described by (15)

$$I \ddot{\theta} + \left[ n + \frac{1}{2} i \omega \pi \rho R^4 d (\Delta f_1 + i \Delta f_2) \right] \dot{\theta} + c \theta = \Gamma_0 e^{i\omega t}, \tag{48}$$

where  $I$  is the moment of inertia of the empty oscillator,  $c$  is the torsion constant, and  $n$  represents damping from non-hydrodynamic sources. If the change in resonant frequency, due to the fluid, is small and if the quality factor of the oscillator is high, it follows from (48) that the resonant frequency  $\nu_R$  is

given by (15)

$$v_R - v_0 = -\frac{\pi v_0 \rho R^4 d \Delta f_1}{4I}, \quad (49)$$

where

$$v_0 = \frac{1}{2\pi} \left( \frac{c}{I} \right)^{1/2}$$

is the resonant frequency of the empty cell. As is obvious, the resonant frequency of the torsional oscillator is proportional to  $\phi_m$ , which is given in (6). The torsional oscillator is another probe for detecting the occurrence of the Fredericks transition in ferroelectric liquid crystals.

The hydrodynamic equations of a thin cell of Sm- $C^*$  in the presence of electric field have been solved. The components of the velocity field have also been obtained in (41) and (42). In our previous work (16), we calculated the hydrodynamic equation of nematic liquid crystals in a magnetic field,  $H$ . In that work, we obtained the resonant frequency that was proportional to  $\phi_m^2$ , whereas here we have obtained the resonant frequency proportional to  $\phi_m$ . This difference comes from the fact that in nematic liquid crystals the free energy is proportional to  $(\hat{\mathbf{n}} \cdot \mathbf{H})^2$ , while in ferroelectric liquid crystals the free energy is proportional to  $\mathbf{P} \cdot \mathbf{E}$ .

## References

- (1) Garoff S.; Meyer R.B. *Phys. Rev. Lett.* **1977**, *38*, 848–851.
- (2) Coleman D.; Mueller D.; Clark N.A.; Maclennan J.E.; Shao R.F.; Bardon S.; Walba D.M. *Phys. Rev. Lett.* **2003**, *91*, 175505–(1–4).
- (3) Poulin P.; Stark H.; Lubensky T.C.; Weitz D.A. *Science* **1997**, *275*, 1770–1773.
- (4) Mieda Y.; Furutani K. *Phys. Rev. Lett.* **2005**, *95*, 177801–(1–4).
- (5) Carlsson T. *Liq. Cryst.* **2001**, *28*, 889–896.
- (6) Zimmermann W.; Ried S.; Pleiner H.; Brand H.R. *Europhys. Lett.* **1996**, *33*, 521–526.
- (7) Carlsson T.; Stewart I.W.; Leslie F.M. *Liq. Cryst.* **1991**, *9*, 661–678.
- (8) Zou Z.; Clark N.A.; Carlsson T. *Jpn. J. Appl. Phys.* **1995**, *34*, 560–565.
- (9) Leslie F.M.; Stewart I.W.; Nakagawa M. *Mol. Cryst. Liq. Cryst.* **1991**, *198*, 443–454.
- (10) de Gennes P.G.; Prost J. *The Physics of Liquid Crystals*; Clarendon Press: Oxford, 2003.
- (11) McMillan W.L. *Phys. Rev. A* **1973**, *8*, 1921–1929.
- (12) Carlsson T.; Clark N.A.; Zou Z. *Liq. Cryst.* **1993**, *15*, 461–477.
- (13) Carlsson T.; Leslie F.M.; Clark N.A. *Phys. Rev. E* **1995**, *51*, 4509–4525.
- (14) Shahzamanian M.A. *J. Phys. Condens. Matter.* **1995**, *7*, 6833–6845.

- (15) Hook J.R. *J. Low Temp. Phys.* **1989**, *74*, 19–44.
- (16) Shahzamanian M.A.; Ghafari M. *Mol. Cryst. Liq. Cryst.* **2004**, *414*, 11–25.
- (17) Blake G.I.; Leslie F.M. *Liq. Cryst.* **1998**, *25*, 319–327.
- (18) Galerne Y.; Martinand J.L.; Durand G.; Veyessie M. *Phys. Rev. Lett.* **1972**, *29*, 562–564.
- (19) Leslie F.M.; Gill S.P.A. *Ferroelectrics* **1993**, *148*, 11–24.

## Appendix A

In this appendix we introduce the 10 coefficients  $\alpha_i$  as follows:

$$\alpha_1 = \frac{1}{2} [\mu_3 + \mu_4 + \lambda_2], \quad (A1)$$

$$\alpha_2 = \frac{1}{2} [\mu_0 + \mu_4 + 2\lambda_2 + \lambda_5], \quad (A2)$$

$$\alpha_3 = \frac{1}{2} [\lambda_2 + \lambda_5], \quad (A3)$$

$$\alpha_4 = \frac{1}{2} [\mu_0 + \mu_5 + 2\lambda_2 - \lambda_3 + \lambda_5 + \lambda_6], \quad (A4)$$

$$\alpha_5 = \frac{1}{2} [\mu_0 + \mu_2 + \mu_4 + \mu_5 - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6], \quad (A5)$$

$$\alpha_6 = \frac{1}{2} [\mu_0 + \mu_4 - 2\lambda_2 + \lambda_5], \quad (A6)$$

$$\alpha_7 = [\lambda_2 - \lambda_5], \quad (A7)$$

$$\alpha_8 = [\mu_0 - \lambda_1], \quad (A8)$$

$$\alpha_9 = \frac{1}{2} [\mu_4 + \mu_5 + 2\lambda_2 - 2\lambda_3 + \lambda_5 + \lambda_6], \quad (A9)$$

$$\alpha_{10} = [\mu_4 - \lambda_2]. \quad (A10)$$

Some of these coefficients has been introduced before (17); for example, measurements for  $\lambda_5$  are available and so are combinations (18). They are compared with data results in (19).